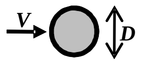
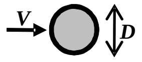
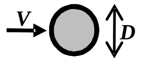
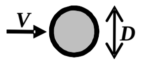
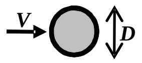
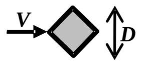
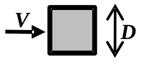
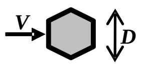
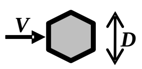
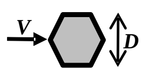
External Flow Correlations (Average, Isothermal Surface)

Flat Plate Correlations

Flow Conditions	Average Nusselt Number	Restrictions
Laminar	$\overline{Nu_L} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.6$
Turbulent	$\overline{Nu_L} = \left(0.037\,\mathrm{Re}_L^{4/5} - A\right)\mathrm{Pr}^{1/3}$ where $A = 0.037\,\mathrm{Re}_{x,c}^{4/5} - 0.664\,\mathrm{Re}_{x,c}^{1/2}$	$0.6 \leq \mathrm{Pr} \leq 60$ $\mathrm{Re}_{x,c} \leq \mathrm{Re}_L \leq 10^8$

Note: All fluid properties are evaluated at film temperature for flat plate correlations.

Cylinders in Cross Flow

Cylinder Cross Section	Reynolds Number Range	Average Nusselt Number	Restrictions
	0.4-4	$\overline{Nu_D} = 0.989\,\mathrm{Re}_D^{0.330}\,\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.7$
	4-40	$\overline{Nu_D} = 0.911\,\mathrm{Re}_D^{0.385}\,\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.7$
	40-4,000	$\overline{Nu_D} = 0.683\,\mathrm{Re}_D^{0.466}\,\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.7$
	4,000-40,000	$\overline{Nu_D} = 0.193\,\mathrm{Re}_D^{0.618}\,\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.7$
	40,000-400,000	$\overline{Nu_D} = 0.027\,\mathrm{Re}_D^{0.805}\,\mathrm{Pr}^{1/3}$	$\mathrm{Pr} \geq 0.7$
	6,000-60,000	$\overline{Nu_L} = 0.304\,\mathrm{Re}_D^{0.59}\,\mathrm{Pr}^{1/3}$	gas flow
	5,000-60,000	$\overline{Nu_D} = 0.158\,\mathrm{Re}_D^{0.66}\,\mathrm{Pr}^{1/3}$	gas flow
	5,200-20,400	$\overline{Nu_D} = 0.164\,\mathrm{Re}_D^{0.638}\,\mathrm{Pr}^{1/3}$	gas flow
	20,400-105,000	$\overline{Nu_D} = 0.039\,\mathrm{Re}_D^{0.78}\,\mathrm{Pr}^{1/3}$	gas flow
	4,500-90,700	$\overline{Nu_D} = 0.150\,\mathrm{Re}_D^{0.638}\,\mathrm{Pr}^{1/3}$	gas flow

Note: All fluid properties are evaluated at film temperature for cylinder in cross flow correlations.

Alternative Correlations for Circular Cylinders in Cross Flow:

- The Zukauskas correlation (7.53) and the Churchill and Bernstein correlation (7.54) may also be used

Freely Falling Liquid Drops

Average Nusselt Number
$\overline{Nu_D} = 2 + 0.6\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}$

Note: All fluid properties are evaluated at T_∞ for the falling drop correlation.

Flow Around a Sphere

Average Nusselt Number	Restrictions
$\overline{Nu_D} = 2 + \left(0.4\,\mathrm{Re}_D^{1/2} + 0.06\,\mathrm{Re}_D^{2/3}\right)\mathrm{Pr}^{0.4}\left(\dfrac{\mu}{\mu_s}\right)^{1/4}$	$0.71 \leq \mathrm{Pr} \leq 380$ $3.5 \leq \mathrm{Re}_D \leq 7.6\times10^4$ $1.0 \leq (\mu/\mu_s) \leq 3.2$

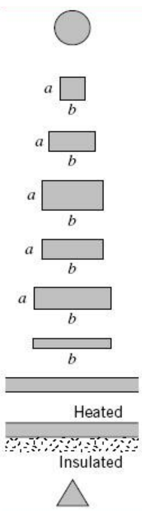
Note: For flow around a sphere, all fluid properties, except μ_s, are evaluated at T_∞. μ_s is evaluated at T_S.

Internal Flow Correlations (Local, Fully Developed Flow)

Note: For all local correlations, fluid properties are evaluated at T_m.
For average correlations, fluid properties are evaluated at the average of inlet and outlet T_m.
If the tube is much longer than the thermal entry length, average correlation ≈ local correlation.

Laminar Flow in Circular and Noncircular Tubes

$$Nu_D \equiv \frac{hD_h}{k}$$



Cross Section	$\frac{b}{a}$	Uniform Heat Flux (Nu_D)	Uniform Surface Temperature (Nu_D)	$f\,\mathrm{Re}_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
Heated / Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Turbulent Flow in Circular Tubes

Local Nusselt Number	Restrictions
$Nu_D = 0.023\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n$	$0.6 \le \mathrm{Pr} \le 160$
$n = 0.40 \; for \; T_s > T_m$	$\mathrm{Re}_D \ge 10{,}000$
$n = 0.30 \; for \; T_s < T_m$	$(L/D) \ge 10$

Turbulent Flow in Noncircular Tubes

For turbulent flow in noncircular tubes, D in the table above may be replaced by $D_h = 4A_c / P$

Alternative Correlations for Turbulent Flow in Circular Tubes:

- The Sieder and Tate Correlation (8.61) is recommended for flows with large property variations
- Another alternate correlation that is more complex but more accurate is provided by Gnielinski (8.62).

Liquid Metals, Turbulent Flow, Constant T_s

Local Nusselt Number	Restrictions
$Nu_D = 5.0 + 0.025 Pe_D^{0.8}$ $Pe_D = \mathrm{Re}_D\,\mathrm{Pr}$	$Pe_D \ge 100$

Note: Only use the correlation in the box directly above for liquid metals. The other correlations on this page are not applicable to liquid metals.

Combined Internal/External Flow Correlations (Average)

Tube banks and packed beds have characteristics of both internal and external flow. The flow is internal in that the fluid flows inside the tube bank/packed bed, exhibits exponential temperature profiles of the mean temperature, and has heat transfer governed by a log mean temperature difference. The flow is external in that it flows over tubes/packed bed particles and that the characteristic dimension in the Reynolds number is based on tube/particle diameter.

Tube Bank Correlation

Average Nusselt Number	Restrictions
$\overline{Nu}_D = C\,\mathrm{Re}_{D,\max}^m\,\mathrm{Pr}^{0.36}\left(\dfrac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4}$	$N_L \geq 20$ $0.7 \leq \mathrm{Pr} \leq 500$ $10 \leq \mathrm{Re}_{D,\max} \leq 2\times10^6$

Note: For tube banks with fewer than 20 rows, multiply the average Nusselt number from the table at left by the correction factor C_2 in Table 7.6. This correction is valid if $\mathrm{Re}_{D,\max}$ is > 1,000.

Configuration	$Re_{D,\max}$	C	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single (isolated) cylinder	
Staggered	10^2–10^3	Approximate as a single (isolated) cylinder	
Aligned $(S_T/S_L > 0.7)$[a]	10^3–2×10^5	0.27	0.63
Staggered $(S_T/S_L < 2)$	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered $(S_T/S_L > 2)$	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

Packed Bed Correlation

Average Nusselt Number	Restrictions
$\varepsilon\,\overline{j}_H = \varepsilon\,\overline{j}_M = 2.06\,\mathrm{Re}_D^{-0.575}$ where $\overline{j}_H = \dfrac{\overline{h}}{\rho V c_p}\mathrm{Pr}^{2/3}$ $\overline{j}_M = \dfrac{\overline{h}_m}{V}Sc^{2/3}$	$\mathrm{Pr}\,(or\ Sc) \approx 0.7$ $90 \leq \mathrm{Re}_D \leq 4{,}000$

External Free Convection Correlations (Average, Isothermal)

Evaluate all fluid properties at the film temperature $T_f = (T_\infty + T_s) / 2$.

Vertical Plate, Vertical Cylinder, Top Side of Inclined Cold Plate, Bottom Side of Inclined Hot Plate

Average Nusselt Number	Restrictions
$\overline{Nu_L} = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}} \right\}^2$	Vertical plate: no restrictions Vertical cylinder: $\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}}$ Top Surface of Inclined Cold Plate / Bottom Surface of Inclined Hot Plate: • Replace g with g cos θ in Ra_L • Valid for $0 \leq \theta \leq 60$

Alternative Correlation for Vertical Plate:

- Equation (9.27) is slightly more accurate for laminar flow.

Horizontal Plate

Orientation	Average Nusselt Number	Restrictions
Upper surface of hot plate or lower surface of cold plate	$\overline{Nu_L} = 0.54 Ra_L^{1/4}$	$10^4 \leq Ra_L \leq 10^7$, $\Pr \geq 0.7$
	$\overline{Nu_L} = 0.15 Ra_L^{1/3}$	$10^7 \leq Ra_L \leq 10^{11}$, all Pr
Lower surface of hot plate or upper surface of cold plate	$\overline{Nu_L} = 0.52 Ra_L^{1/5}$	$10^4 \leq Ra_L \leq 10^9$, $\Pr \geq 0.7$

$$L \equiv \frac{A_s}{P}$$

Curved Shapes

Shape	Average Nusselt Number	Restrictions
Long Horizontal Cylinder	$\overline{Nu_D} = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}} \right\}^2$	$Ra_D \leq 10^{12}$
Sphere	$\overline{Nu_D} = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1 + (0.469/\Pr)^{9/16}\right]^{4/9}}$	$\Pr \geq 0.7$ $Ra_D \leq 10^{11}$

Alternative Correlation for Long Horizontal Cylinder:

- The Morgan correlation (9.33) may also be used.

Internal Free Convection Correlations

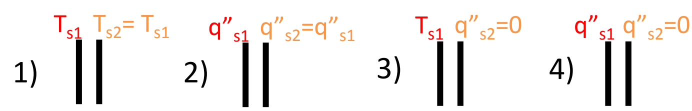
Vertical Parallel Plate Channels (Developing and Fully Developed)

Boundary Condition	Nusselt Number	Rayleigh Number	Getting q and q_s'' from Nu	Temperature to evaluate fluid properties in Ra
isothermal (T_s known on one or both plates)	Average Nu over whole plate $\overline{Nu_S} = \left[\frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}}\right]^{-1/2}$	$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\alpha\nu}$	$\overline{Nu_S} = \left(\frac{q/A}{T_s - T_\infty}\right)\frac{S}{k}$	$\overline{T} = \frac{T_s + T_\infty}{2}$
isoflux (q_s'' known on one or both plates)	Local Nu at x = L $Nu_{S,L} = \left[\frac{C_1}{Ra_S^* S/L} + \frac{C_2}{(Ra_S^* S/L)^{2/5}}\right]^{-1/2}$	$Ra_S^* = \frac{g\beta q_s'' S^4}{k\alpha\nu}$	$Nu_{S,L} = \left(\frac{q_s''}{T_{s,L} - T_\infty}\right)\frac{S}{k}$	$\overline{T} = \frac{T_{s,L} + T_\infty}{2}$

S = plate spacing; T_∞=inlet temperature (same as ambient); $T_{s,L}$=surface temperature at x=L

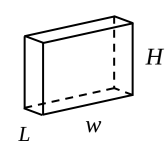
	Surface Condition	C_1	C_2
1)	Symmetric isothermal plates ($T_{s,1} = T_{s,2}$)	576	2.87
2)	Symmetric isoflux plates ($q''_{s,1} = q''_{s,2}$)	48	2.51
3)	Isothermal/adiabatic plates ($T_{s,1}, q''_{s,2} = 0$)	144	2.87
4)	Isoflux/adiabatic plates ($q''_{s,1}, q''_{s,2} = 0$)	24	2.51

C_1 and C_2 are given for four different sets of surface boundary conditions. Use the isothermal equation for conditions 1 and 3; isoflux equation for conditions 2 and 4.



Vertical Rectangular Cavity

Average Nusselt Number	Restrictions
$\overline{Nu_L} = 0.18\left(\frac{\Pr}{\Pr + 0.2} Ra_L\right)^{0.29}$	$1 \le (H/L) \le 2$; $10^{-3} \le \Pr \le 10^5$; $10^3 \le \frac{Ra_L \Pr}{0.2 + \Pr}$
$\overline{Nu_L} = 0.22\left(\frac{\Pr}{\Pr + 0.2} Ra_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$	$2 \le (H/L) \le 10$; $\Pr \le 10^5$; $10^3 \le Ra_L \le 10^{10}$
$\overline{Nu_L} = 0.42 Ra_L^{1/4} \Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3}$	$10 \le (H/L) \le 40$; $1 \le \Pr \le 2\times10^4$; $10^4 \le Ra_L \le 10^7$



Alternative Correlation for Vertical Rectangular Cavity:

- Eq. (9.53) covers a wide range of aspect ratios but is more restrictive on Ra and Pr

Horizontal Cavity Heated From Below

Average Nusselt Number	Restrictions
$\overline{Nu_L} = 0.069 Ra_L^{1/3} \Pr^{0.074}$	$3\times10^5 \le Ra_L \le 7\times10^9$

For cavity correlations, evaluate all fluid properties at the average surface temperature $\overline{T} = (T_1 + T_2)/2$. L is the distance between hot and cold walls.

Correlations for Inclined/Tilted Geometries:

- Inclined parallel plate channels: (9.47)
- Tilted rectangular cavities: (9.54)-(9.57)

Correlations for Curved Geometries:

- Space between concentric horizontal cylinders: (9.58)
- Space between concentric spheres: (9.61)

Boiling and Condensation

Nucleate Pool Boiling

$$q_s'' = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}\Delta T_e}{C_{s,f}h_{fg}\,\Pr_l^n}\right)^3$$

Evaluate liquid and vapor properties at T_{sat} .

Surface–Fluid Combination	$C_{s,f}$	n
Water–copper		
Scored	0.0068	1.0
Polished	0.0128	1.0
Water–stainless steel		
Chemically etched	0.0133	1.0
Mechanically polished	0.0132	1.0
Ground and polished	0.0080	1.0
Water–brass	0.0060	1.0
Water–nickel	0.006	1.0
Water–platinum	0.0130	1.0
n-Pentane–copper		
Polished	0.0154	1.7
Lapped	0.0049	1.7
Benzene–chromium	0.0101	1.7
Ethyl alcohol–chromium	0.0027	1.7

Critical Heat Flux

$$q''_{\max} = C h_{fg}\rho_v\left[\frac{\sigma g(\rho_l-\rho_v)}{\rho_v^2}\right]^{1/4}$$

Evaluate liquid and vapor properties at T_{sat} .

C =0.149 for large horizontal plates.
C=0.131 for large horizontal cylinders, spheres, and many large finite heated surfaces.

Film Boiling

$$\overline{Nu}_D = \frac{\bar{h}_{conv}D}{k_v} = C\left[\frac{g(\rho_l-\rho_v)h'_{fg}D^3}{\nu_v k_v(T_s-T_{sat})}\right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.80c_{p,v}(T_s-T_{sat})$$

Evaluate vapor properties at $T_f = (T_{sat} + T_s) / 2$.
Evaluate ρ_l and h_{fg} at T_{sat} .

C =0.67 for spheres. C=0.62 for horizontal cylinders.

Radiation should be considered for $T_s > 300°C$
See Eqs. (10.9)-(10.11)

Correlations for Flow Boiling:

- External forced convection boiling: (10.12)-(10.14)
- Two-phase flow: (10.15)-(10.16)

For all condensation correlations below:

Evaluate liquid properties at $T_f = (T_{sat} + T_s) / 2$.
Evaluate ρ_v and h_{fg} at T_{sat} .

Laminar Film Condensation, Vertical Flat Plate

$$\bar{h}_L = 0.943\left[\frac{\rho_l g(\rho_l-\rho_v)h'_{fg}k_l^3}{\mu_l(T_{sat}-T_s)L}\right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68c_{p,l}(T_{sat}-T_s)$$

Laminar, Transition, and Turbulent Film Condensation, Vertical Flat Plate (for $\rho_l >> \rho_v$):

- Calculate the parameter P using (10.42), then solve for h_L using the appropriate correlation from (10.43)-(10.45)

Film Condensation, Vertical Tube:

- Vertical flat plate expressions can be used if δ(L) << D/2. Evaluate δ(L) using (10.26).

Laminar Film Condensation, Sphere and Tube

$$\bar{h}_D = C\left[\frac{\rho_l g(\rho_l-\rho_v)h'_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68c_{p,l}(T_{sat}-T_s)$$

C =0.826 for spheres.

C=0.729 for horizontal tubes.

Laminar Film Condensation, Vertical Tier of N Tubes:

- Average heat transfer coefficient of each tube: Eq. (10.49).

Inner Surface of Horizontal Tube

Average Nusselt Number	Restrictions
$\bar{h}_D = 0.555\left[\frac{\rho_l g(\rho_l-\rho_v)h'_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}$ $h'_{fg} = h_{fg} + 0.375c_{p,l}(T_{sat}-T_s)$	$\left(\frac{\rho_v u_{m,v}D}{\mu_v}\right)_i < 35{,}000$
Eq. (10.51)	$\left(\frac{\rho_v u_{m,v}D}{\mu_v}\right)_i \geq 35{,}000$

Dropwise Condensation

Average Nusselt Number	Restrictions
$\bar{h}_{dc} = 51{,}104 + 2044T_{sat}(°C)$	$22°C \le T_{sat} \le 100°C$
$\bar{h}_{dc} = 255{,}510$	$T_{sat} \ge 100°C$